The Social Multiplier of Tax Evasion: Evidence from Italian Audit Data

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Abstract - We investigate the role of individual interdependencies in tax evasion, arising from congestion on the auditing resources available to local tax authorities. Identification exploits a novel method based on comparison of the variance of individual behavior — concealed income in this case — at different levels of aggregation, within different subpopulations (Graham, 2008). This method allows us to mitigate some of the most severe problems that surround identification of neighbourhood effects, at the cost of identifying restrictions that arise naturally from our model. We employ a unique dataset of tax audits to about 75,000 self-employed individuals in Italy. Surprisingly, this sample is not statistically different from a random sample of taxpayers. We find a social multiplier of about 3, meaning that the equilibrium response to a shock that induces an exogenous variation in mean concealed income — such as tougher or looser tax enforcement — is about three times the initial average response.


Keywords: social interactions, social multiplier, tax evasion, tax compliance, excess variance

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Theft—whether from the state, from a fellow citizen or from a looted Jewish store—was so widespread that in the eyes of many people it ceased to be a crime.

—Tony Judt, *Postwar*.1

## 1 Introduction

Tax evasion is an endemic phenomenon in virtually any economic system. Economists have developed two main theoretical approaches to explain it.2 The first, initiated by Allingham and Sandmo (1972) and Yitzhaky (1974), models taxpayers as amoral expected utility maximizers: rewards and punishments associated with truthful and untruthful income reports provide extrinsic motivations for behavior. A shortcoming of the basic version of this approach is that it predicts too much tax evasion relative to what is normally observed in advanced economies. The second approach obviates this difficulty by introducing the idea of “tax morale”, i.e. an intrinsic motivation inducing people to abide by their tax obligations [e.g. Gordon (1989).]

Even in this different guise, the standard approach can hardly accommodate evidence of large differences in tax compliance between units of observation with comparable monetary incentives and arguably similar values or intrinsic motivations.3 Consider Italy, for instance. In this country tax evasion is at top levels among OECD countries, but the picture varies greatly from region to region and even within regions, which are fairly homogeneous units.4 Such heterogeneity, however, is *not* at odds with possible

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2 For a summary of theoretical models and evidence see the two exhaustive surveys by Andreoni et al. (1998) and Slemrod (2007).
3 For instance, the World Values Survey asks individuals all over the world whether it is justifiable to cheat on taxes or not, on a scale ranging from 1 (never justifiable) to 10 (always justifiable). The means in 1999 were 2.3 in the US and Spain, 2.4 in the UK, Portugal, and Italy, 2.6 in Switzerland (Swiss figures refer to 1996). Yet, according to estimates of the size of the shadow economy (an imperfect indicator of tax evasion, although one that is available for most countries) reported by Schneider (2005), the percentage of GDP that went unreported, on average, between 2002 and 2003 was about 9% in the US and Switzerland, 12% in the UK, 22% in Spain and Portugal, 27% in Italy.
4 Pisani and Polito (2006) estimate that the ratio between concealed and reported income from productive activities across Italian regions, on average between 1998 and 2002, ranges from 13% in Lombardy and 22% in Emilia Romagna and Veneto, to 66% in Sicily and 94% in Calabria. The variance across provinces within regions is noticeable. Just to mention the extremes, in Lombardy the ratio ranges from 5% in the province of Milan to 34% in the neighboring province of Lodi. In Calabria, it ranges from 53% (Reggio
homogeneity in fundamentals if there are local externalities in the decision to cheat on taxes. An important source of such externalities is suggested by economic models of crime [Sah (1991)]: the perceived probability of being caught and punished decreases if more people cheat on taxes and only a fixed numbers of individuals can be audited. In fact this is an implication of models of tax compliance where the taxpayers and the tax authority interact strategically [e.g. Sánchez and Sobel (1993)], but one that has not been brought to data yet.\textsuperscript{5}

The goal of this paper is to identify externalities generated by congestion in auditing resources in a simple structural model of tax evasion. In our model local tax authorities optimally choose an audit policy, given a certain amount of auditing resources, in order to induce risk-neutral taxpayers to report a level of taxable income as close as possible to the true level. In equilibrium, the perceived individual probability of being audited decreases with the extent of tax evasion in the jurisdiction. This is so because in the short run the tax authority’s budget is given.

There are of course other examples of social externalities in tax evasion. First, tax cheating is an activity that requires the development of particular skills, and people may learn from others how to conceal their income. Second, cheating on taxes may clash with social norms whose strength decreases with the extent of tax evasion itself [Myles and Naylor (1996)]. Third, informality of businesses can be transmitted across the production chain if value added taxes based on the credit-liability system are in use [de Paula and Scheinkman (2008)].\textsuperscript{6}

These are all are examples of neighborhood effects: decision makers influence each other directly within certain reference groups [Durlauf (2004)]. For this reason they are also referred to as endogenous social effects [Manski (1993); Brock and Durlauf (2001a)]. These imply the existence of a social multiplier: small exogenous changes in fundamentals – such as tougher or looser tax enforcement – cause relatively large aggregate responses. The reason is that an exogenous shock affects the level of tax evasion directly via individuals’ private incentives and indirectly via social interactions. Thus, the latter can generate relatively large variation in tax evasion across units

\textsuperscript{5}This same mechanism is employed in recent theoretical work on the optimal design of tax schemes in the presence of equilibria characterized by high tax evasion (Bassetto and Phelan, 2007).

\textsuperscript{6}Interestingly, this possibility is consistent with the low levels of unreported GDP in the US and Switzerland (see footnote 3), where value-added taxes are not in use and in use at a very low rate, respectively.
with similar fundamentals.

We focus on the congestion channel for two reasons. First, it is a parsimonious and testable way of modelling local interdependencies and to formalize the idea that widespread tax evasion generates a sense of invulnerability in tax cheaters. Second, and most important from our viewpoint, it allows us to define reference groups in a very natural way, thus avoiding arbitrary assumptions about who interacts with whom: in our model this is defined by the audit system itself. Of course other kind of externalities may be at work, an issue on which we will return at the end of the paper.

Our modelling assumptions fit well the nature of our dataset. This is a unique collection of about 75,000 tax audits to self-employed individuals (small businesses and professionals) performed in Italy at the beginning of the 1990s, and become final. The audits were performed by local branches of the fiscal authority, who can rely on a given amount of resources and have jurisdiction on a precise geographic area. We show that, somewhat surprisingly, this sample is not statistically different from a random sample.

Identification exploits a novel method, due to Graham (2008), based on comparison of variances of individual behavior at different levels of aggregation within distinct subpopulations. The method is able to overcome a number of well-know difficulties in empirical studies of social interactions, notably the bias stemming from group unobservables. Furthermore, being very parsimonious in terms of data requirements, this method is particularly apt when dealing with scant administrative datasets like the one we work with. We find significantly positive externalities that imply a large social multiplier: the equilibrium aggregate response to an exogenous shock that affects average reported income is about three times the initial response.

To our knowledge, the only empirical work that attempts the measurement of social effects in tax evasion is Fortin, Lacroix and Villemal (2007). This experimental study is complementary to ours for both the kind of data the authors use as well as the estimation method. The main advantage of their approach is that it allows genuinely random audit probabilities. On the other hand, in addition to problems of external validity, lab experiments on tax evasion can be regarded as akin to surveys, as experimental subjects are effectively asked to report to the experimenter how much they evade. The problem is that, as shown in previous research, in surveys about tax compliance there is virtually no correlation between reported and actual tax evasion (Ellfers et al., 1987). This is consistent with the zero or even negative effect Fortin, Lacroix and Villemal find. The most important advantage of our approach is that it is based on audit data that, in the light of our model, provide a natural definition of reference groups. Furthermore, our estimatio
method allows for arbitrary individual- and group-level heterogeneity.

The paper develops as follows. In the next Section we provide basic intuition into why variances at different levels of aggregation reflect social interactions, and how they can help identification. In Section 3 we anticipate the description of our dataset: this will facilitate the construction and illustration of the theoretical model, presented in Section 4. The model naturally leads to the econometric framework, which we present in Section 5 along with an illustration of how identification is achieved. In Section 6 we report and discuss our results. Section 7 concludes. Most derivations are relegated in an Appendix.

2 Variance and Social Interactions

Our work, as well as the identifying procedure it relies on, are closely related to the pioneering study of Glaeser, Sacerdote, and Scheinkman (1996), henceforth GSS, on crime and social interactions. This study was motivated by evidence of high variance of crime across observationally similar cities in the US. As we mentioned above, a puzzling aspect of tax evasion common to other types of illegal activities is the remarkable variance across countries as well as across regions within countries. A quick inspection of our data (which we describe in more detail below) reveals that the distribution of tax evasion across tax jurisdictions in Italy is characterized by large dispersion even after conditioning on regions, which are quite homogeneous units. This information is summarized in Figures 1a, 1b and 1c, where we plot the conditional distributions of the share of taxable income that was concealed.

High variance may of course simply reflect heterogeneity in fundamentals, for instance institutional and individual characteristics, at the local level. The question is how much such differences can explain. The novel insight of GSS was that when the observed cross-group variation of individual behavior exceeds what can be accounted for by fundamentals (observable and unobservable), one is left with “excess variance”. In a linear model, as we illustrate in detail below, such residual variance is entirely due to the presence of endogenous social interactions among decision makers. This is intuitive: complementarities among individual choices generate feedbacks.

7 There are marked differences across Italian regions with respect to market, cultural, and social conditions, as well as tax attitudes. Some of these differences have deep historical roots, tracing back to the turbulent process of national unification in the XIX Century. In particular Southern Italy (where levels of tax evasion are extremely high) was literally conquered by rulers of Piedmont – the future royal family. In this part of the country, taxes were naturally regarded as contributions to an occupying state.
that amplify the effect of changes in fundamentals on individual behavior. Straightforward as it sounds, the problem is how to detect excess variance in practice in the presence of unobservable individual and group characteristics. Formally, the identification problem is how to distinguish within-group correlation that is due to social interactions, from within-group correlation that is due to other common causes at the individual- and group-level. For instance, when trying to detect social influences in tax evasion, one cannot control for the efficiency or corruptibility of local tax officials, nor for the level of tax morale. In the context of their empirical application, GSS “admit that there is considerable uncertainty as to how much of the cross-city variance is actually explained by urban characteristics” (p. 509). Similar identification problems arise if one looks at individuals rather than larger units (Manski, 1993; Moffitt, 2001). In these circumstances, one needs identifying assumptions. GSS took the following route:

In this paper we face an identification problem that can be solved either with an assumption about the functional form of unobserved heterogeneity or by assuming a “reasonable” level of that heterogeneity. Unfortunately, the reader may not find either assumption plausible, in which case our empirical work must be seen as identifying the magnitude of some combination of social interactions and unobserved heterogeneity. (p. 517)

The identification route we take in this paper, due to Graham (2008), develops the framework of GSS into a more powerful statistical method, although essentially restricted to a linear world. The central insight of this method is that social interactions amplify the between-group variance of individual behavior with respect to the within-group variance. Comparing such variances within distinct subpopulations allows to distill the portion of cross-group variation that is due to social interactions only. This is possible as long as one of the dimensions defining subpopulations does not affect certain components of the covariance matrix of individual behavior, which can be conveniently interpreted as reflecting sorting of individuals across groups, matching between individuals and group-level attributes, and the variance of the latter. In other words, one needs an instrument that influences the between-group variance only through the within-group variance, i.e. an exogenous variation.8 Our model suggests such an instrument. The

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8This property is not confined to variances: Manacorda (2006) uses the ratio between effects at different levels of aggregation (within- and between-families in this case) to identify intra-family externalities in children’s propensity to enter the labor market.
reason why we believe this method is more powerful than the one originally used by GSS is that it is robust to arbitrary unobservable heterogeneity, it works under milder identifying assumptions, and so is able to considerably mitigate the “unfortunately” part in the above quotation.

3 Data

Before illustrating the model and the identification strategy, we briefly describe the data we use. This anticipation will facilitate the illustration of our modelling strategy and identifying assumptions.

3.1 A sample of tax audits

We employ a cross-section of ordinary tax audits concerning about 75,000 self-employed individuals in Italy. These are essentially small individual businesses and professionals. The audits were performed by the Italian Guardia di Finanza (Tax Police, a special force dependent on the Ministry of the Economy and Finance that is in charge of tax audits in Italy) at the beginning of the 1990s and are now final, after all possible appeals. Therefore, our dataset is the universe of tax audits in a certain period. Individuals belong to local tax jurisdictions. These are defined by the sphere of competence of local branches of the Ministry of the Economy and Finance, which are responsible for tax management and audits. There are about 400 tax jurisdictions in Italy: each of them comprises one or more municipalities, or a portion of a large city.

In addition to such geographic information, we observe reported income for the purposes of personal income tax (Imposta sul Reddito delle Persone Fisiche, IRPEF), as well as the amount found by the audit. We take the latter to be an accurate estimate of actual income. We also observe the sector of economic activity, tax deductions (allowances) and tax credit. For the last two variables we know the amount claimed and the amount the taxpayer was actually entitled to according to the tax authority. Under the Italian tax code, tax allowances typically include deductions for compulsory contributions to pension funds and charitable deductions. On the other hand, a tax credit is typically granted for dependents, and for special personal expenses (a certain percentage of health expenses and mortgage interests). Moreover, and quite crucially for our identification strategy, there are de facto distinct audit processes for taxable income and tax credits. Since the main component of the latter consists of a credit for dependents with low income, the tax authority can easily verify whether a taxpayers claimed the correct tax
credit or not. Such checks are performed routinely and essentially at no cost by simply cross-checking information reported by the taxpayer with family information at the register and dependents’ income in the tax authority’s database. This is a fundamental difference with audits on taxable income, which are instead performed only on a small fraction of cases, as reported in Table 1.\textsuperscript{9} Consistently with this fact, we observe that while in our dataset only one in four individuals correctly reports taxable income, as many as 90% correctly report the tax credit they are entitled to.

Tables 1-3 report summary statistics. Tax evasion turns out to be remarkably high among Italian self-employed that were audited: only 26% of them correctly reported taxable income, and concealed income (the difference between actual and reported taxable income) is more than 45% the average taxable income. However, there is considerable variability in fiscal non-compliance among different categories of taxpayers across regions and sectors of economic activity. In general we observe higher rates of non-compliance in central and southern regions and among taxpayers employed in agriculture and commerce and among craftsmen.

3.2 Is this a random sample?

Tax audits, of course, are not random: tax authorities audit taxpayers on the basis of precise criteria, and we should expect tax cheaters to be oversampled. These criteria were not made public in Italy at the time our data refer to. If these were known, we could construct sampling weights and base our estimates on consistent estimators of population moments: this could be enough to correct for this basic form of sample selection.\textsuperscript{10} Since we cannot do this, one may wonder whether we can draw any credible inference.

We claim that our sample is not too far from a random sample, and we offer two pieces of evidence.

First, Schneider (2005) estimates that at the end of the 1990s the size of the shadow economy in Italy was about 23% of GDP. We want to compare this figure to what we observe in our sample. In order to do so, we must correct the ratio to take into account the we work with a sample of self-employed individuals. Ideally, we would like to know what is the percentage of the shadow economy in the self-employment sector. This is very hard to assess, but we can adjust Schneider’s estimate to get closer to the number of interest. We must first exclude the fraction of GDP that originates in the

\textsuperscript{9}We are grateful to Giorgio Zanella for directing our attention to such an important institutional difference.

\textsuperscript{10}See Pfefferman (1993) and references therein.
public sector: this is an innocuous adjustment of the denominator because by definition this portion cannot originate in the informal sector. In the early 1990s, this was about 28% in Italy (20% from consumption expenditure by general government, and 8% from collective consumption). Next, Schneider’s definition of the shadow economy explicitly excludes illegal activities. However, according to recent estimates, large criminal organizations in Italy\textsuperscript{11} may account for an astonishing 15% of GDP. At least part of this ends up in official statistics, because in Italy they are corrected for the black economy. A conservative estimate is 2%. By correspondingly reducing the measure of GDP in Schneider’s ratio, we make numerator and denominator consistent. Therefore, even without further adjustments\textsuperscript{12}, for comparison with our sample Schneider’s figure is $0.23/(1 - 0.28 - 0.2) \approx 33\%$. In our data, the corresponding measure is the fraction of total income (i.e. taxable plus non-taxable income) that goes unreported. This is 37%. We believe that the difference between these two numbers should be much larger if tax cheaters were considerably over-represented in our sample.

Second, and consistently with this belief, in our data the distribution of economic activities and the distribution of tax audits across these same sectors are statistically indistinguishable, in all regions. We performed Kolmogorov-Smirnov tests by region, and we can never reject the hypothesis that the two distributions are equal. The smallest $p$-value is 0.83, with the majority above 0.99.

These two pieces of evidence suggest that our sample, somewhat surprisingly, is not far from a random sample of taxpayers. Our interpretation is that this reflects the inherent difficulties tax authorities face in detecting tax cheaters. That is, although auditing activities are designed to select individuals who are \textit{ex-ante} more likely to evade taxes – a key implication of theoretical models of optimal enforcement – the outcome is \textit{ex-post} not too far from random sampling.

4 The Model

Our theoretical framework follows by and large Sánchez and Sobel (1993). Consider a population of $N$ taxpayers, indexed by $i = 1, ..., N$. Consistently with our data, we take these to be individual producers distributed across

\textsuperscript{11}The estimates we refer to aggregate Mafia, Camorra, 'Ndrangheta, and Sacra Corona Unita.

\textsuperscript{12}The fraction of employment and corporate income that originates in the informal sector is likely to be considerably smaller than the corresponding fraction of self-employment income [Slemrod (2007)]
G groups, indexed by $g = 1, ..., G$. Each group has fixed size, denoted $N_g$, with boundaries defined by the jurisdiction of local tax authorities, who are in charge of tax enforcement. Auditing resources available to tax authority $g$ amount to $A_g$. Taxpayers are allowed to move across jurisdictions, as we model below. A taxpayer $i$ whose taxable income is $y_i$ (i.e. gross income minus tax allowances the individual is entitled to) reports an amount $y^R_i$ to the local tax authority, facing a probability-of-audit schedule $p_{ig}(y^R_i)$, and pays a tax at an exogenous flat rate $t$ on reported income. Taxable income is private information and the tax rate is exogenously determined. The tax authority can audit income at the cost of $a > 0$ per audit. When an individual is audited, the tax authority observes taxable income. If $y^R_i \geq y_i$, i.e. at least the due tax was paid, nothing happens. If $y^R_i < y_i$, i.e. income was partly concealed, the residual due tax must be paid and a fine is imposed on the unpaid tax, at an exogenous rate $f > 0$. Furthermore, the taxpayer deducts a tax credit from due tax. We denote with $c_i$ the total tax credit taxpayer $i$ is entitled to, and with $c^R_i$ the amount he claims, facing a probability-of-audit schedule $q_{ig}(c^R_i)$ and a fine rate $\phi > 0$. Consistently with our data, we treat the two audit policies as separate, and we assume that only income audits are costly.

The taxpayer’s problem is to choose how much taxable income to report, given the audit policies for taxable income and the tax credit, so to minimize expected payment. The tax authority’s problem is to choose such audit policies, given its budget and the optimal behavior of the taxpayer, so to maximize expected revenue within its jurisdiction. The expectation is taken with respect to the marginal distributions of income and tax credit, denoted $F$ and $H$, conditional on individual and jurisdiction-specific information, denoted $\chi_i$ and $\Upsilon_g$ respectively. We can write the taxpayer’s problem as follows: \footnote{We implicitly condition on truthful or under-reporting, because over-reporting is strictly dominated, for instance, by truthful reporting.}

\[
\begin{aligned}
\min_{y^R_i, c^R_i} & : (1 - p_{ig}) y^R_i + tp_{ig} (y_i - f (y_i - y^R_i)) \\
& - (1 - q_{ig}) c^R_i - q_{ig} (c_i - \phi (c^R_i - c_i)) \\
\text{subject to } & : p_{ig} = p^*_{ig}(y^R_i) ; q_{ig} = q^*_{ig}(c^R_i),
\end{aligned}
\]

where $p^*_{ig}(y^R_i)$ and $q^*_{ig}(c^R_i)$ are the optimal audit probabilities chosen by the tax authority, i.e. solve:
\[
\max_{\{p_{ig}\}_{i=1}^{N_g}, \{q_{ig}\}_{i=1}^{N_g}} \sum_{i=1}^{N_g} \left[ (1 - p_{ig}) ty_i^R + tp_{ig} \int_0^\infty \left( y + f (y - y_i^R) \right) dF (y | \chi_i, \Upsilon_g) 
- (1 - q_{ig}) c_i^R - q_{ig} \int_0^\infty \left( c - \phi (c_i^R - c) \right) dH (c | \chi_i, \Upsilon_g) \right]
\]

subject to: \[ \sum_{i=1}^{N_g} ap_{ig} \leq A_g. \]

In other words, the tax authority cannot exceed its budget, in terms of expected number of audits. Since the cost of auditing the claimed tax credit is zero, it is optimal to set \( q_{ig} = 1 \). As a consequence, the taxpayer truthfully reports the tax credit, or \( c_i^R = c_i \). Both these facts replicate what we observe in our data, as illustrated in the previous Section. As a consequence, the tax credit is uninformative for the tax authority.

At an interior optimum, the first-order necessary conditions for a minimum of the taxpayer's objective balance the expected marginal cost and benefit of under-reporting income and over-reporting tax credits, respectively, net of the marginal effect on the probability of an audit:\(^\text{14}\)

\[
fp_{ig}^* (y_i^R) = (1 - p_{ig}^* (y_i^R)) + \frac{\partial p_{ig}^* (y_i^R)}{\partial y_i^R} (1 + f) (y_i - y_i^R).
\]

As for the optimal \( p_{ig} \), the tax authority can always elicit a truthful report by setting \( p_{ig} = (1 + f)^{-1} \).\(^\text{15}\) However, in any interesting situation the budget constraint is binding. In this case, Sánchez and Sobel (1993) show that under mild regularity conditions the tax authority optimally divides taxpayers into at most three income classes. Our model differs from theirs because we allow for rich individual and group-level heterogeneity – this is needed for the empirical analysis. However, the nature of the solution is the same because of the additive form of the tax authority’s objective. In this case, income classes are conditional on individual and group information: if taxpayer \( i \) in jurisdiction \( g \) reports income below a threshold \( y_{ig} \), he is audited with probability \( (1 + f)^{-1} \); if he reports above a threshold \( y_{ig} \), he is not audited; if he reports an amount in-between he is audited with some constant probability \( \pi_{ig} \). This is illustrated in Figure 1.

\(^\text{14}\)As long as \( \partial p_{ig}/\partial y_i^R \leq 0 \), and \( \partial q_{ig}/\partial c_i^R \leq 0 \), which makes intuitive sense and will in fact be the case in equilibrium, the second order condition for a minimum is satisfied.

\(^\text{15}\)For any taxpayer, this is the value of \( p_{ig} \) that balances expected income under truthful reporting and expected income under tax evasion.
The level of the thresholds, and so of $\pi ig$, are of course endogenous and depend on the other model parameters. In particular, since at the optimum $p_{ig}$ is non increasing in $y_{Ri}$ and the resource constraint is binding, as marginal taxpayers – i.e. taxpayers at the right of the two thresholds – reduce reported income the probability that they are audited increases, and so the probability that the other taxpayers in the jurisdiction are audited must decrease. This is best seen if we rewrite the budget constraint of the tax authority at the optimum:

$$\sum_{i=1}^{N} \frac{1}{1 + f} \mathbb{I} \left[ \bar{y}_{Ri} < y_{ig} \right] + \sum_{i=1}^{N} \pi_{ig} \mathbb{I} \left[ \underline{y}_{ig} \leq y_{Ri} < \bar{y}_{ig} \right] \leq \frac{A_{g}}{a}.$$  

where $\mathbb{I} \left[ \cdot \right]$ is the indicator function, equal to one if the proposition inside the brackets is true and zero otherwise. Therefore, $p_{ig}$ must be increasing in the amount reported by other taxpayers, a vector denoted $y_{-i}$. This is the fundamental externality we want to stress. Figure 2, below, illustrates: given true income, when everybody except taxpayer $i$ in the jurisdiction conceals more, and auditing resources are given, the tax authority becomes more lenient towards taxpayer $i$. More generally, the presence of such externality allows to write the optimal audit probability for any taxpayer $i$ implicitly as follows:

$$p_{ig}^{*} = p \left( y_{Ri}, y_{-Ri}, \chi_{i}, \chi_{-i}, t, f, \Upsilon_{g}, A_{g} \right) ,$$

Figure 1. Optimal audit probability.
where variables indexed by $-i$ contain information about all individuals in jurisdiction $g$ except $i$. This expression clarifies in what sense a model à la Sánchez and Sobel (1993) generates neighborhood effects in tax evasion.

**Figure 2. Effect of increased tax evasion**

![Figure 2](image)

Although taxpayers behave optimally, we do not assume that they know the functional form, let alone the value, of this probability function: tax authorities do not disclose their exact auditing criteria, and even expert researchers have a hard time figuring them out. Like in the pioneering model of Sah (1991), we simply assume the taxpayer acts optimally on the basis of a perceived, or inferred, probability of audit. We allow taxpayers to behave like econometricians and use the best linear predictor:

$$
\hat{p}_i = b_0 + b_1 y^R_i + b_2 y^R_g + b_3 \chi_i + b_4 Z_g,
$$

where $y^R_g$ is average reported income in jurisdiction $g$, $Z_g \equiv (\chi_g, \Upsilon_g, A_g)$ collects group-level, contextual, variables, $\chi_g$ is the group-level averages of $\chi_i$, and the coefficients are generated by an hypothetical linear projection. The use of group-level means reflects the idea that by observing what is happening in the jurisdiction, or in communicating with other taxpayers, an individual can inexpensively estimate “average” conditions around him, while it would be too costly to collect specific information about any single
taxpayer in the group.\textsuperscript{16} Our model imposes restrictions on the sign of $b_1$ and $b_2$: the first is negative, and the second is positive. Figure 3 shows such linear prediction function and its shift following an increase in local tax evasion.

Figure 3. Optimal and optimally predicted audit probability

We next illustrate key implications of the model, which we will exploit for identification. First, the optimal audit policy defines a value function for the tax authority, denoted $R(A_g)$, i.e. the maximum expected revenue raised in location $g$. Suppose a revenue-maximizing central government allocates resources to local tax authorities. It will do so by solving the following problem:

$$\max_{\{A_g\}} \sum_{g=1}^{G} R(A_g), \text{ s.t. } \sum_g A_g \leq A,$$

where $A$ denotes an exogenous amount of resources the central government allocates to tax enforcement. Denote with $c_g$ the jurisdiction average tax credit. Since the optimal audit policy is characterized by (4) and $q_{ig}^*(c_i^R) = 1$, inspection of problem (6) allows to establish the following:

**Proposition 1** Allocation of auditing resources is independent of $c_g$.\textsuperscript{16}

\textsuperscript{16}Casual conversations with self-employed individuals suggest that they have a fairly accurate perception of how much tax evasion there is in their area.
The reason is of course that the tax credit is uninformative for the tax authority. Second, we allow individuals to sort across jurisdictions for the purpose of minimizing expected payment. Since the tax rate is constant, this is equivalent to minimizing the probability of being audited. That is, ignoring mobility costs:

\[ \min_{g} p_{ig}^{*}, \quad \text{s.t.} \quad g \leq G. \]  

(7)

By inspection of this problem and equation (4), one can establish the following — for the same reason underlying Proposition 1:

**Proposition 2** Sorting of taxpayers across jurisdictions is independent of \( c_{g} \).

Claims 1 and 2, in turn, imply the following:

**Proposition 3** Contextual variables in \( Z_{g} \) are independent of \( c_{g} \).

**Proposition 4** Matching between the contextual variables in \( Z_{g} \) and taxpayer’s characteristics is independent of \( c_{g} \).

Finally, we illustrate how this framework leads to the estimating equation. In the Appendix we show that replacing the predicted, or perceived, probability of an audit (5) into the taxpayer’s first-order condition (2), the latter boils down to the well-known linear-in-means behavioral equation of Manski (1993):

\[ e_{i} = \beta X_{i} + \delta Y_{g} + Je_{g}, \]  

(8)

where \( \beta, \delta, \) and \( J \) are coefficients, \( X_{i} \equiv (1, y_{i}, \chi_{i}) \), \( Y_{g} \equiv (y_{g}, Z_{g}) \), and \( e_{g} \) and \( y_{g} \) are group-averages of the corresponding individual-level variables. Equation (8) is a reaction-function, and \( J \) is the main parameter of interest, because it measures externalities across taxpayers. In Manski’s (1993) terminology it captures endogenous social interactions, since \( e_{g} \) is endogenously determined. This is the parameter we want to estimate. As per derivation, all of the parameters in (8) admit a structural interpretation. In particular, as shown in the Appendix, \( J \equiv -b_{2}/2b_{1} > 0 \). This is the slope of optimal individual concealed income with respect to the group-level average, and reflects the distinct and opposite effects of individual \( (b_{1}) \) and average \( (b_{2}) \) reported income on the perceived probability of an audit. Such probability
decreases with average concealed income, and so it is optimal to conceal less income if others report more. The individual effect, in turn, dampens the social incentive to report more, because the more an individual reports the lower the perceived probability of an audit. The last step needed to solve the model is to explicitly compute average concealed income in jurisdiction $g$: $e_g = \frac{1}{N_g} \sum_i e_i = \beta X_g + \delta Y_g + J e_g$, and so

$$e_g = \gamma \beta X_g + \gamma \delta Y_g, \quad (9)$$

where $X_g$ is the jurisdiction-level mean of $X_i$, and $\gamma \equiv (1 - J)^{-1}$. The model has a unique Nash equilibrium characterized by the following individual level of concealed income, obtained by replacing (9) into (8):

$$e_i = \beta X_i + (\gamma - 1) \beta X_g + \gamma \delta Y_g, \quad (10)$$

since $J \gamma \equiv \gamma - 1$. Parameter $\gamma$ is the social multiplier. Notice that stability requires $J \leq 1$, otherwise concealed income would explode following a tiny shock. Therefore, $\gamma \geq 1$. If $\gamma = 1$, i.e. $J = 0$, there are no externalities, and others’ characteristics ($X_g$) are irrelevant in equilibrium. The social multiplier is the limit of the series of feedbacks triggered by an exogenous variation in contextual or average individual characteristics within a group. If such exogenous variation generates an initial, i.e. before the feedbacks associated with social interactions, 1% change in average concealed income and $J > 0$, then in equilibrium, i.e. after feedbacks are exhausted, the variation is $\gamma \%$, which is larger than the initial response. Effectively, the social multiplier is a sort of “autoelasticity” of average behavior.\(^{17}\)

Suppose that the tax authority gives taxpayers to understand that it will be looser in enforcing norms against evasion, so that each individual is encouraged to conceal $x\%$ more. When people realize that tax evasion actually increased, they perceive a lower probability of detection and optimally conceal more, which further increases the average, and so on. The final increase in average concealed income is $\gamma x\%$. On the contrary, suppose the government wishes to reduce tax evasion, and targets a reduction of $z\%$ in average concealed income. Then it is enough to increase auditing resources just enough to induce an average reduction of just $(z/\gamma)\%$.

\(^{17}\)Equivalently, it is the ratio between the average cumulative response and the initial individual response following an exogenous shock (see Glaeser, Sacerdote, and Scheinkman, 2003). In this sense, although it might look inappropriate to use an absolute measure of tax evasion rather than a relative measure (e.g. the share of income that goes unreported), it is not obvious how to interpret the social multiplier in the second case (because in relative terms, average evasion is not the mean of the individual levels)
5 Identification

The inferential problem is to estimate the endogenous social interactions coefficient, i.e. $J$ in structural equation (8), and the implied social multiplier, $\gamma$. The reason why identification is a concern can be illustrated as follows. Equations (8) and (9) form a system of simultaneous equations. Identification of the first equation, and so of $J$, requires an exclusion restriction in the form of a variable that affects average but not individual concealed income. This is the essence of the ‘reflection problem’ [Manski (1993)], i.e. the problem of separating the effect of mutual influences ($\gamma$) from the effect of common influences (contextual and correlated effects $\delta$ and $\beta$) in the reduced form, equation (10). Since the same contextual controls, $Y_g$, necessarily appear in both equations, such a restriction must consist of an individual effect whose average is not a contextual effect, that is a variable in $X_g$ that is not in $Y_g$ [Brock and Durlauf (2001b)]. Even if such an instrument were available there are at least two additional difficulties. First, since individuals sort across jurisdictions, problem (7), all of the group-level variables, not only average concealed income, are endogenous. Second, a number of unobservable group-level effects work as confounding factors. For instance, how does one control for the efficiency and corruptibility of local tax officials? Such unobserved group effect would be wrongly interpreted as the effect of social interactions.

The method devised by Graham (2008) allows to considerably mitigate all of these problems. Let us consider, in addition to individual concealed income $e_i$, the mean tax credit $c_g$, and a vector $W_g$ containing information that is appropriate for the problem at hand – as we describe below.

Next, we rewrite the equilibrium equation (10) in error-component form, assuming that anything except $e_i$, $c_g$, and $W_g$ is observable. Define $\alpha_g \equiv dY_g$ as group-level heterogeneity (after purging it from $W_g$, an innocuous abuse of notation), $\varepsilon_i \equiv cX_i$ as individual-level heterogeneity18, and $\varepsilon_g \equiv cX_g$ as the group-level average of the latter. Then equation (10) and (9) become:

\[
\begin{align*}
    e_i &= \gamma \alpha_g + \varepsilon_i + (\gamma - 1) \varepsilon_g, \quad (11) \\
    e_g &= \gamma (\alpha_g + \varepsilon_g). \quad (12)
\end{align*}
\]

Equation (11) is slightly different from Graham’s behavioral equation, where group-level heterogeneity, $\alpha_g$, is not amplified by the social multiplier,

---

18This way of modeling individual heterogeneity makes our model consistent with theories allowing for different types of taxpayers motivations.
γ. Such lack of amplification exclude that shocks to contextual variables may trigger a chain of feedbacks between group and individual behavior, which is instead the case in our framework.19 The reason is that equation (11) is derived from an economic model in which altering institutional variables leads to changes in behavior that are subsequently propagated by social interactions. If one posits a behavioral equation in which interactions occur directly via average individual characteristics (rather than indirectly in the reduced form, like in our model), then contextual variables that are not captured by average individual characteristics cannot generate multiplier effects. Correspondingly, Graham (2008) recognizes that in his framework

\[ \gamma \] may be a composite function of multiple ‘structural’ parameters. In Manski (1993) it depends on the strength of what he terms ‘exogenous’ and ‘endogenous’ social effects. (p. 5, footnote 6).

Thanks to the explicit modeling of the mechanism generating social interactions, the social multiplier we identify has a sharper interpretation, i.e. it is a known function of structural parameters that reflects endogenous social interactions only. Therefore it is the exact measure of the equilibrium effect of exogenous shocks that alter individual behavior.

Let us denote by \( \sigma^2 (c_g, W_g) \) the variance of individual heterogeneity, conditional on the mean tax credit and other information in \( W_g \), by \( \sigma_{\epsilon\epsilon} (c_g, W_g) \) the corresponding conditional covariance across individuals, by \( \sigma^2 (c_g, W_g) \) the conditional variance of group-level heterogeneity, and by \( \sigma_{\epsilon\alpha} (c_g, W_g) \) its conditional covariance with individual level heterogeneity. Notice that \( \sigma_{\epsilon\epsilon} \) is a measure of the degree of sorting of taxpayers across jurisdictions: it should be zero if they were randomly assigned. Similarly, \( \sigma^2 \) reflects the variance of unobserved characteristics of the tax authority (such as the efficiency of tax officials and the resources they can rely on), as well as other institutional, cultural, or market characteristics common to all taxpayers in a jurisdiction. Finally, \( \sigma_{\epsilon\alpha} \) reflects the degree of matching between such characteristics and taxpayers. This covariance is non-zero when, for instance, resources are allocated to tax authorities on the basis of taxpayers’ characteristics, or when taxpayers sort across jurisdictions on the basis of the efficiency of the tax authority. Propositions 1, 2 and 4 allow us to establish the following:

19For instance, an exogenous change in the quality of tax officials in a certain jurisdiction may trigger a change in the perception of the audit probabilities and a sequence of feedbacks between individuals thus leading to a different equilibrium level of tax evasion.
Proposition 5 \( \sigma^2 \alpha(c_g, W_g), \sigma_{\alpha\varepsilon}(c_g, W_g), \) and \( \sigma_{\varepsilon\varepsilon}(c_g, W_g) \) are all independent of \( c_g \).

In other words, the variation in the quality of tax officials and other relevant contextual and institutional characteristics, as well as the matching of tax officials with taxpayers and sorting of the latter across jurisdictions, while possibly dependent on the variables included in \( W_g \) are not affected by the mean tax credit. This is true in our model because the tax credit contains no useful information to detect tax evasion. In fact the correlation between individual tax evasion and individual tax credit in our data is extremely low (0.01). Proposition 5 provides a restriction that allows identification,\(^{20}\) as we illustrate next following Graham (2008) closely.

Equations (11) and (12) express concealed income at different levels of aggregation. We show in the appendix that after conditioning on \( c_g \) and \( W_g \) and using Proposition 5, the within-group variance of concealed income in jurisdiction \( g \), denoted \( V^w_g \), and the corresponding between-group variance, denoted \( V^b_g \), can be written as follows:

\[
V^w_g = \mathbb{E} \left[ \frac{\sigma^2 (c_g, W_g) - \sigma_{\varepsilon\varepsilon} (W_g)}{N_g} \right]_{c_g, W_g} \tag{13}
\]

\[
V^b_g = \gamma^2 (\sigma^2 (W_g) + 2\sigma_{\alpha\varepsilon} (W_g) + \sigma_{\varepsilon\varepsilon} (W_g) + V^w_g) \tag{14}
\]

These conditional variances exhibit an important pattern. First, the within-group variance is independent of social interactions and group-level heterogeneity. This is quite intuitive. For instance, if there are differences in individual tax evasion in a jurisdiction where tax officials are corrupt or people influence each other, such variability of course cannot be ascribed to corruption or interactions, but only to differences between individuals and to covariance between individual characteristics generated by the process of sorting. Second, the between-group variance depends on group heterogeneity and is amplified by social interactions, when these are present (i.e. \( \gamma > 1 \)). This is also intuitive. For instance, part of the variability of tax evasion between two groups, one where tax officials are corrupt and one where they are not, must depend on corruption, as well as on the fact that dishonest

\(^{20}\)One may still be concerned about people sorting across jurisdictions on the basis of personal characteristics summarized by the tax credit. For instance, families with many children may tend to live in the same residential areas. This remains an issue, but we notice that it is considerably mitigated by the fact that the size of jurisdictions in our data is large enough to encompass entire cities or large portions of them.
taxpayers may tend to locate where tax officials are more easily corruptible. However, since the level of tax evasion in a group depends on social interactions, which alters contextual differences, so must be for cross-group variation. This was exactly the original intuition of Glaeser, Sacerdote, and Scheinkman (1996): the presence of social interactions generates a wedge between the variance of illegal behavior at different levels of aggregation. In our model the size of the wedge is proportional to $\gamma^2$, a fact that offers a lever for identification.

If we assume that the portion of the between-group variance that is independent of the within-group variance can be written as a linear function of $W_g$, i.e.

$$
\gamma^2 \left( \sigma^2_{\alpha \alpha} (W_g) + 2\sigma_{\alpha \varepsilon} (W_g) + \sigma_{\varepsilon \varepsilon} (W_g) \right) = \theta W_g, \quad (15)
$$

and if we rewrite variances as expectations of appropriate statistics $G^u_{\omega}$ and $G^b_{\omega}$ (see appendix), i.e.

$$
V^w_g \equiv \mathbb{E}\left(G^u_{\omega}|c_{g},W_g\right), \quad (16)
$$

$$
V^b_g \equiv \mathbb{E}\left(G^b_{\omega}|c_{g},W_g\right), \quad (17)
$$

then (14) becomes:

$$
\mathbb{E}\left(G^u_{\omega}|c_{g},W_g\right) = \theta W_g + \gamma^2 \mathbb{E}\left(G^b_{\omega}|c_{g},W_g\right). \quad (18)
$$

This equation generates a conditional moment restriction:

$$
\mathbb{E}\left[G^b_{\omega} - \theta W_g - G^u_{\omega}|c_{g},W_g\right] = 0, \quad (19)
$$

which in turn implies the following unconditional moment restriction:

$$
\mathbb{E}\left[\left(\begin{array}{c} c_g \\ W_g \end{array}\right) \left(G^b_{\omega} - \theta W_g - \gamma^2 G^u_{\omega}\right)\right] = 0. \quad (20)
$$

The latter provides the basis for estimating $\gamma^2$ by GMM. The average tax credit works as an instrument: Proposition 5 implies that it affects the between-group variance of concealed income only through the within-group variance, as equations (13) and (14) illustrate, i.e. provides an exogenous variation. The reason why it influences the within-group variance – a rank
condition we can test\footnote{More precisely, the condition is:} – is that it affects the variance of individual heterogeneity.\footnote{Remember that in our sample a tax credit is granted for things such as dependents, health expenses, mortgage interests. At the same time, in our data the average tax credit is essentially uncorrelated with mean taxable income and mean concealed income, which reinforces our confidence in the quality of the instrument.} Effectively, $c_g$ restricts the covariance matrix of cross-group tax evasion. Therefore, as well illustrated by Durlauf and Tanaka (2008), such a covariance restriction parallels the exclusion restriction needed to identify social interactions in a regression framework based on model (8).

This identification strategy is robust to arbitrary individual and group-level unobservables. Under our identifying assumptions, unobservables may arbitrarily affect average tax evasion, as well as the within- and cross-group variances. What matters is the relation between the two, equation (18): we know that the slope, a function of the wedge between variances at different levels of aggregation, identifies the social multiplier, regardless of whether unobservables affect the level of this function, or the intercept.

By writing the unconditional moment restriction like in (20) we are not addressing the issue of the optimal set of instruments, so our estimates will be consistent but not necessarily efficient.\footnote{Given the conditional moment restriction (19), there exist infinite functions $\phi (w_g)$ such that $\mathbb{E} \left[ \phi (w_g) \left( G_{y}^{\mu} - \beta w_{1g} - \gamma^2 G_{y}^{\mu} \right) \right] = 0$.} Feasibility requires an estimate of $G^\mu_{y}$: we use the predicted value from the regression of $e_i$ on a constant, $c_g$, all the variables in $W_g$, and their squares. After identifying $\gamma^2$, the delta method can then be used to recover the structural parameter $J$ – the endogenous social interactions coefficient – as well as its standard error.\footnote{Notice that the model (that is, equation (18)) is linear and is exactly identified. Therefore, the GMM estimator of $\gamma^2$ associated with the moment restrictions in (20) is identical to the linear instrumental variables estimator.}

6 Results

Our discussion above suggests to include in $W_g$ information that may affect sorting, matching, and allocation of auditing resources. The following information seems appropriate given the limitations of our dataset and the finding that the distribution of tax audits across sectors is indistinguishable from random distribution. One, an indicator for whether the jurisdiction is large or small relative to the regional median. Two, an indicator for whether
the local share of aggregate taxable income that is reported is above or below the regional share. For instance, more efficient tax officials may be assigned to larger jurisdictions, as well as to jurisdictions that do not generate enough revenue relative to the region. Three, a South dummy: Table 1 indicates that tax evasion is much more relevant in Southern than in Northern Italy. It is quite intuitive that this additional variable may affect at least matching and allocation of resources.

Our results are reported in Table 4 below. We estimate, across two alternative specifications, an endogenous social interaction coefficient ranging from 0.65 to 0.69, which implies a social multiplier ranging from 2.9 to 3.2. Column 1 contains our baseline specification, without regional control. Column 2 adds regional information. The first-stage $F$-statistic and the coefficient on the instrument at first stage indicate that the instrument secures identification. However, the $F$-statistic may raise concerns about weak identification. In order to deal with these residual concerns, we follow the simple strategy suggested by Angrist and Krueger (2001) and look at the reduced form equations. The coefficient on the instrument in the reduced form is reported towards the end of Table 4 (Reduced form: Coefficient on $c_g$). This must be significantly different from zero to dispel the presumption that there are no social interactions or that the instrument is weak. This is in fact the case in both specifications.

### Table 4. Results.

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<tr>
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<tbody>
<tr>
<td></td>
<td>(robust s.e.)</td>
<td>(robust s.e.)</td>
</tr>
<tr>
<td>$J$</td>
<td>0.69</td>
<td>0.65</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>(0.08)</td>
<td>(0.11)</td>
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**1st stage:**

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<tr>
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<tbody>
<tr>
<td>$F$-stat</td>
<td>5.41</td>
<td>5.54</td>
</tr>
<tr>
<td>p-value</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>Coefficient on $c_g$</td>
<td>18.14</td>
<td>26.34</td>
</tr>
<tr>
<td>(robust s.e.)</td>
<td>(7.80)</td>
<td>(11.26)</td>
</tr>
</tbody>
</table>

**Reduced form:**

<table>
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<tr>
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<tbody>
<tr>
<td>Coefficient on $c_g$</td>
<td>186.90</td>
<td>220.63</td>
</tr>
<tr>
<td>(robust s.e.)</td>
<td>(83.65)</td>
<td>(108.87)</td>
</tr>
</tbody>
</table>

Regional control: no, yes
Jurisdictions: 412, 412
Individuals: 78,863, 78,863
These estimates imply a strong amplifying role of social interactions: an exogenous shock altering concealed income independently across individuals, produces an equilibrium variation that is about three times the initial response. This strong effect reflects the importance of reported income in the jurisdiction with respect to the perceived probability of an audit: recall the structural expression for the social interactions coefficient, $J = -b_1/2b_2$, where $b_1$ and $b_2$ are the effects of own and average report, respectively, on the perceived probability of an audit. Our estimates imply that, in absolute value, $b_2$ is 30-40% larger than $b_1$: for a dishonest taxpayer, doing business in a jurisdiction where other taxpayers tend to report high income is perceived to be more risky than reporting low income, which is suggestive of the importance of social externalities.

7 Concluding Remarks

Despite social externalities constitute a plausible explanation for the high variation of tax compliance, empirical research on tax evasion has largely ignored this possible determinant of individual behavior. This lack of empirical research is largely due to the fact that social interactions are extremely difficult to identify. In this paper we have exploited a method recently devised by Graham (2008) that allows to overcome some of the most worrying aspects of identification, namely group unobservables and self-selection. In our application, this “variance method” requires restrictions on the covariance matrix of concealed income that parallel the assumptions needed to identify social interactions based on the traditional “regression method” (Durlauf and Tanaka, 2008). Such restrictions arise naturally in our application thanks to an institutional property of the tax audit system in Italy: contrary to income reports, claims to a tax credit are routinely and costlessly audited.

An important difference between the two tests is that the variance method, being based on group-level data, is more parsimonious in terms of data requirements and so mitigates the problem of individual and group unobservables, as well as self-selection, that affect the regression method. We regard these as important advantages. Thanks to linearity, which follows from risk-neutrality of individual entrepreneurs and their attempt to optimally forecast audit probabilities, we are able to provide a simple structural interpretation of our estimates. The driving force is the externality deriving from the resource constraint of local tax authorities. Two remarks are necessary about this assumption.
First, it may sound odd to refer to such interdependencies as social interactions. However, ‘social’ refers to the particular nature of these externalities, which affect only individuals belonging to the same group, regardless of whether the forces that generate the externalities are genuinely ‘social’ or more ‘technological’, like in our model. In either case the meaning and the policy implications of the multiplier effects we identify are unaffected.

Second, we focus on only one of the various mechanisms that may generate social interdependency in tax evasion. Of course our empirical investigation remains valid whatever the mechanism that generates salience of social interactions in tax evasion. At worst, our estimate reflects the compound effect of different forces operating at the level of tax jurisdictions. Only the structural interpretation of the social multiplier is tied to the particular mechanism we focus on.

Conditional on our identifying assumption, our work suggests a social multiplier of about 3: reducing the extent of tax evasion is less costly than it is commonly thought. From a methodological viewpoint, we regard our work as an important example of how the social interactions literature can make concrete progress in an empirical direction.

References


Appendix

8.1 Derivation of equation (8)

Replace first $\partial \hat{p}^*_i / \partial y^R_i = b_1$, from equation (5), into (2). Then the latter becomes:

$$fp_{ig} = (1 - p_{ig}) + b_1 (1 + f) e_i,$$

or, equivalently, collecting terms and dividing both sides by $(1 + f)$:

$$p_{ig} = (1 + f)^{-1} + b_1 e_i.$$

Next, replace (5) into this expression. From the definition of concealed income, we can also replace $y^R_i \equiv y_i - e_i$ and $y^R_g \equiv y_g - e_g$ and obtain:

$$b_0 + b_1 y_i - b_1 e_i + b_2 y_g - b_2 e_g + b_3 t_i + b_4 \chi_i + b_5 Z_g = (1 + f)^{-1} + b_1 e_i.$$

Solving for $e_i$ yields:

$$e_i = \frac{b_0 - (1 + f)^{-1}}{2b_1} + \frac{b_4}{2b_1} \chi_i + \frac{b_3}{2b_1} t_i + \frac{1}{2} y_i + \frac{b_5}{2b_1} Z_g + \frac{b_2}{2b_1} y_g - \frac{b_2}{2b_1} e_g.$$
Defining $k \equiv \frac{b_0-(1+f)}{2b_1}$, $\beta \equiv \left( \frac{b_4}{2b_1}, \frac{b_2}{2b_1} \right)$, $\delta \equiv \left( \frac{b_5}{2b_1}, \frac{b_3}{2b_1} \right)$, $J \equiv -\frac{b_2}{2b_1}$, and $Y_g \equiv (Z_g, y_g)$, this equation is the same as (8).

### 8.2 Derivation of conditional moments (16) and (17)

For each group $g$ that comprises $N_g$ individuals in the population of interest, we have a random sample of size $n_g \leq N_g$. Denote with $e_g^s$ the sample mean of concealed income in group $g$, and manipulate the data in terms of within-group ($w$) and between-group ($b$) deviations from the respective means, with the cross-group mean conditioned on observable group-level information, that is $c_g$ and $W_g$:

\[
G_w^g \equiv \frac{1}{N_g} \frac{1}{n_g-1} \sum_{i=1}^{n_g} (e_i - e_g^s)^2, \quad (21)
\]

\[
G_b^g \equiv (e_g^s - \mathbb{E}(e_i|c_g, W_g))^2 - \left( \frac{1}{n_g} - \frac{1}{N_g} \right) N_g G_w^g. \tag{22}
\]

In words, $G_w^g$ is simply the within-group sample variance of tax evasion, normalized by population size, while $G_b^g$ is the square deviation of group average evasion in the sample from the conditional population mean, minus a correction term to account for the discrepancy between sample and population means. The role of this correction term is made clear below. The purpose of these statistics is to derive an estimator based on the variance of concealed income at different levels of aggregation. Notice that after using individual-level data to construct $G_w^g$ and $G_b^g$, the analysis uses only these transformations and $(c_g, W_g)$, that is aggregate group-level data: jurisdictions, not individual taxpayers, are the units of observation.

The normalized conditional within-group variance of concealed income, within the jurisdictions defined by $(c_g, W_g)$, can be computed by taking the conditional expectation of (21), as expressed in equation (13). In what follows we will use equation (11):

\[
e_i = \gamma \alpha_g + \varepsilon_i + (\gamma - 1) \varepsilon_g.
\]

Assume, without loss of generality, that $\mathbb{E}(\varepsilon_i|N_g, n_g, W_g) = 0$.

\[25\] Notice that this is an assumption about the theoretical mean. The sample mean, as assumed above, is $\bar{e}_g = \frac{1}{n_g} \sum_{i} e_i$. Then, the conditional mean of concealed income is:

\[26\]
\[ E(e|W_g) = \gamma E(\alpha_g|W_g). \]

Taking the conditional expectations of \( G^w_g \) yields:

\[
E[G^w_g | N_g, n_g, W_g, c_g] = E\left[ \frac{1}{N_g n_g - 1} \sum_{i=1}^{n_g} (e_i - \varepsilon_g)^2 | N_g, n_g, W_g, c_g \right]
\]

\[
= E\left[ \frac{1}{N_g n_g - 1} \sum_{i=1}^{n_g} (\gamma \alpha_g + \varepsilon_i + (\gamma - 1) \varepsilon_g - \gamma \alpha_g - \varepsilon_g - (\gamma - 1) \varepsilon_g)^2 | N_g, n_g, W_g, c_g \right]
\]

\[
= E\left[ \frac{1}{N_g n_g - 1} \sum_{i=1}^{n_g} (\varepsilon_i - \varepsilon_g)^2 | N_g, n_g, W_g, c_g \right]
\]

\[
= E\left[ \frac{1}{N_g n_g - 1} \sum_{i=1}^{n_g} (\varepsilon_i^2 + \varepsilon_g^2 - 2\varepsilon_i \varepsilon_g) | N_g, n_g, W_g, c_g \right]
\]

\[
= \frac{n_g}{N_g n_g - 1} E\left( \varepsilon_i^2 | N_g, n_g, W_g, c_g \right) + \frac{n_g}{N_g n_g - 1} E\left( \varepsilon_g^2 | N_g, n_g, W_g, c_g \right)
\]

\[
- 2 \frac{n_g}{N_g n_g - 1} E\left( \varepsilon_i \varepsilon_g | N_g, n_g, W_g, c_g \right)
\]

\[
= \frac{n_g}{N_g n_g - 1} \sigma^2_e + \frac{1}{N_g (n_g - 1)} \sum_{j=1}^{n_g} E\left( \varepsilon_i^2 | N_g, n_g, W_g, c_g \right)
\]

\[
- 2 \frac{1}{N_g (n_g - 1)} E\left( \varepsilon_i \sum_{j=1}^{n_g} \varepsilon_j | N_g, n_g, W_g, c_g \right)
\]

\[
= \frac{n_g}{N_g n_g - 1} \sigma^2_e + \frac{1}{N_g (n_g - 1)} \left( n_g \sigma^2_e + n_g (n_g - 1) \sigma_{ee} \right)
\]

\[
- 2 \frac{1}{N_g (n_g - 1)} \left( \sigma^2_e + (n_g - 1) \sigma_{ee} \right)
\]

\[
= \frac{n_g}{N_g n_g - 1} \sigma^2_e - \frac{1}{N_g (n_g - 1)} \sigma^2_e - \frac{1}{N_g} \sigma_{ee} = \frac{\sigma^2_e - \sigma_{ee}}{N_g},
\]

where we abbreviate \( \sigma^2_e (W_g) \) and \( \sigma_{ee} (W_g) \) with \( \sigma^2_e \) and \( \sigma_{ee} \). By the law of
iterated expectations $\mathbb{E} \left[ G_g^W | W_g \right] = \mathbb{E} \left[ \mathbb{E} \left[ G_g^W | N_g, n_g, W_g \right] | W_g \right]$. That is:

$$
\mathbb{E} \left[ G_g^w | c_g, W_g \right] = \mathbb{E} \left[ \frac{\sigma_g^2 - \sigma_{\varepsilon g}}{N_g} | c_g, W_g \right],
$$

that is equation (13). Similarly, the between-jurisdiction conditional variance can be computed by taking the conditional expectation of (22), as expressed in equation (14):

$$
\mathbb{E} \left[ G_g^B | N_g, n_g, W_g \right] = \mathbb{E} \left[ \left( e_g - E(e_g|\nu, \omega) \right)^2 | N_g, n_g, W_g \right] - \mathbb{E} \left[ \left( \frac{1}{n_g} - \frac{1}{N_g} \right) N_g G_g^W | N_g, n_g, W_g \right] = \mathbb{V}(e_g|N_g, n_g, W_g) - \mathbb{E} \left[ \left( \frac{1}{n_g} - \frac{1}{N_g} \right) \left( \sigma_g^2 - \sigma_{\varepsilon g} \right) | N_g, n_g, W_g \right],
$$

(23)

where $\mathbb{V}$ denotes variance. Denote with $\varepsilon_g^n$ the sample mean of individual characteristics in group $g$, as opposed to population mean $\varepsilon_g$. Then we can write the first term in this equation, i.e. the between-group conditional variance of concealed income, as:

$$
\mathbb{V}(e_g|N_g, n_g, W_g) = \mathbb{V} \left( \frac{1}{n_g} \sum_{i=1}^{n_g} (\gamma \alpha_g + \varepsilon_i + (\gamma - 1) \varepsilon_g) | N_g, n_g, W_g \right) = \mathbb{V} \left( \gamma \alpha_g + \varepsilon_g^n + (\gamma - 1) \varepsilon_g | N_g, n_g, W_g \right) + \gamma^2 \mathbb{C}(\alpha_g, \varepsilon_g^n|N_g, n_g, W_g) + \gamma (\gamma - 1) \mathbb{C}(\alpha_g, \varepsilon_g | N_g, n_g, W_g) + 2(\gamma - 1) \mathbb{C}(\varepsilon_g^n, \varepsilon_g|N_g, n_g, W_g) + 2(\gamma - 1) \mathbb{C}(\varepsilon_g^n, \varepsilon_g | N_g, n_g, W_g)
$$

$$
= \gamma^2 \sigma_\alpha^2 + \mathbb{V} \left( \frac{1}{n_g} \sum_{i=1}^{n_g} \varepsilon_i | N_g, n_g, W_g \right) + \gamma^2 \mathbb{C}(\varepsilon_i, \varepsilon_i|N_g, n_g, W_g) + 2\gamma \sigma_{\alpha \varepsilon} + 2\gamma (\gamma - 1) \sigma_{\alpha \varepsilon} + 2(\gamma - 1) \mathbb{C} \left( \frac{1}{n_g} \sum_{i=1}^{n_g} \varepsilon_i, \frac{1}{n_g} \sum_{i=1}^{n_g} \varepsilon_i | N_g, n_g, W_g \right)
$$

$$
= \gamma^2 \sigma_\alpha^2 + \frac{1}{n_g} \sigma_\varepsilon^2 + \frac{n_g - 1}{n_g} \sigma_{\varepsilon \varepsilon} + (\gamma - 1)^2 \frac{1}{N_g} \sigma_\varepsilon^2 + (\gamma - 1)^2 \frac{N_g - 1}{N_g} \sigma_{\varepsilon \varepsilon} +
$$
\[ +2\gamma \sigma_{\alpha\epsilon} + 2\gamma (\gamma - 1) \sigma_{\alpha\epsilon} + 2(\gamma - 1) \left( \frac{1}{N_g} \sigma_\epsilon^2 + \frac{N_g - 1}{N_g} \sigma_{\epsilon\epsilon} \right) \]
\[ = \gamma^2 \sigma_\alpha^2 + 2\gamma^2 \sigma_{\alpha\epsilon} + \left( (\gamma - 1)^2 + 2(\gamma - 1) + 1 \right) \sigma_{\epsilon\epsilon} \]
\[ + \frac{\sigma_\epsilon^2 - \sigma_{\epsilon\epsilon}}{n_g} + \left( (\gamma - 1)^2 + 2(\gamma - 1) \right) \frac{\sigma_\epsilon^2 - \sigma_{\epsilon\epsilon}}{N_g} \]
\[ = \gamma^2 \sigma_\alpha^2 + 2\gamma^2 \sigma_{\alpha\epsilon} + \gamma^2 \sigma_{\epsilon\epsilon} + \gamma^2 \frac{\sigma_\epsilon^2 - \sigma_{\epsilon\epsilon}}{N_g} + \left( \frac{1}{n_g} - \frac{1}{N_g} \right) \left( \sigma_\epsilon^2 - \sigma_{\epsilon\epsilon} \right) \]

where we have again, for brevity, omitted the argument of the conditional variances and covariances. Now replace (24) into (23):

\[ = \gamma^2 \left( \sigma_\alpha^2 + 2\sigma_{\alpha\epsilon} + \frac{\sigma_\epsilon^2 - \sigma_{\epsilon\epsilon}}{n_g} + \left( \frac{1}{n_g} - \frac{1}{N_g} \right) \left( \sigma_\epsilon^2 - \sigma_{\epsilon\epsilon} \right) \right) \]
\[ - \mathbb{E} \left[ \left( \frac{1}{n_g} - \frac{1}{N_g} \right) \left( \sigma_\epsilon^2 - \sigma_{\epsilon\epsilon} \right) \mid N_g, n_g, W_g \right] . \]

Finally, take the expectation of this expression, conditional on \( W_g \):

\[ = \gamma^2 \left( \sigma_\alpha^2 + 2\sigma_{\alpha\epsilon} + \sigma_{\epsilon\epsilon} + \frac{\sigma_\epsilon^2 - \sigma_{\epsilon\epsilon}}{N_g} + \gamma^2 \mathbb{E} \left[ \frac{\sigma_\epsilon^2 - \sigma_{\epsilon\epsilon}}{N_g} \mid W_g \right] \right) \]
\[ + \mathbb{E} \left[ \left( \frac{1}{n_g} - \frac{1}{N_g} \right) \left( \sigma_\epsilon^2 - \sigma_{\epsilon\epsilon} \right) \mid W_g \right] \]
\[ - \mathbb{E} \left[ \mathbb{E} \left[ \left( \frac{1}{n_g} - \frac{1}{N_g} \right) \left( \sigma_\epsilon^2 - \sigma_{\epsilon\epsilon} \right) \mid N_g, n_g, W_g \right] \mid W_g \right] . \]

Applying the law of iterated expectations, the LHS reduces to \( \mathbb{E} \left[ G_g^B \mid c_g, W_g \right] \), and the last two terms on the RHS cancel out. Therefore this equation reduces to equation (14).
8.3 Some useful properties of the covariance

Three basic properties of the covariance are used in the previous derivations. We summarize them here for convenience. First, for any sequence of \( n \) random variables \( X_i \) with common variance \( \sigma^2_X \) and covariance \( \sigma_{XX} \):

\[
\mathbb{V} \left( \frac{1}{n} \sum_{i=1}^{n} X_i \right) = \frac{1}{n^2} \left( n\sigma^2_X + 2 \sum_{i<j}^{n} \text{cov}(X_i, X_j) \right) = \frac{1}{n^2} \left( n\sigma^2_X + 2 \frac{n(n-1)}{2} \sigma_{XX} \right) = \frac{1}{n} \sigma^2_X + \frac{(n-1)}{n} \sigma_{XX}.
\]

Second, for the same sequence and another random variable \( Y \) whose covariance with any random variable in the series is \( \sigma_{YX} \):

\[
\mathbb{C} \left( Y, \frac{1}{n} \sum_{i=1}^{n} X_i \right) = \frac{1}{n} \sum_{i=1}^{n} \text{cov}(Y, X_i) = \sigma_{YX}
\]

Third, if we extend the sequence to \( N > n \), then:

\[
\mathbb{C} \left( \frac{1}{n} \sum_{i=1}^{n} X_i, \frac{1}{N} \sum_{i=1}^{N} X_i \right) = \frac{1}{nN} \left( \sum_{i=1}^{n} \text{cov}(X_i, X_i) + \sum_{i, j > n}^{N} \text{cov}(X_i, X_j) \right) = \frac{1}{nN} \left( n\sigma^2_X + (nN - n) \sigma_{XX} \right) = \frac{1}{N} \sigma^2_X + \frac{N-1}{N} \sigma_{XX}
\]
Figure 1a. Distribution (kernel density estimates) of concealed income across jurisdictions, as % of total taxable income
Figure 1b. Distribution (kernel density estimates) of concealed income across jurisdictions, as % of total taxable income
Figure 1c. Distribution (kernel density estimates) of concealed income across jurisdictions, as % of total taxable income
Table 1. Concealed income and tax audits.

<table>
<thead>
<tr>
<th>Region</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aosta Valley</td>
<td>36.8%</td>
<td>2</td>
<td>267</td>
<td>6,852</td>
<td>3.9%</td>
</tr>
<tr>
<td>Piedmont</td>
<td>37.6%</td>
<td>38</td>
<td>6,887</td>
<td>237,068</td>
<td>2.9%</td>
</tr>
<tr>
<td>Lombardy</td>
<td>32.5%</td>
<td>60</td>
<td>12,634</td>
<td>520,765</td>
<td>2.4%</td>
</tr>
<tr>
<td>Friuli-Venezia Giulia</td>
<td>56.3%</td>
<td>10</td>
<td>1407</td>
<td>65,208</td>
<td>2.2%</td>
</tr>
<tr>
<td>Trentino-Südtirol</td>
<td>45.0%</td>
<td>12</td>
<td>1067</td>
<td>50,697</td>
<td>2.1%</td>
</tr>
<tr>
<td>Veneto</td>
<td>50.4%</td>
<td>31</td>
<td>4,840</td>
<td>259,584</td>
<td>1.9%</td>
</tr>
<tr>
<td>Liguria</td>
<td>45.6%</td>
<td>10</td>
<td>3,661</td>
<td>95,602</td>
<td>3.8%</td>
</tr>
<tr>
<td>Emilia-Romagna</td>
<td>37.5%</td>
<td>24</td>
<td>6,056</td>
<td>248,353</td>
<td>2.4%</td>
</tr>
<tr>
<td>Tuscany</td>
<td>40.7%</td>
<td>34</td>
<td>5468</td>
<td>215,758</td>
<td>2.5%</td>
</tr>
<tr>
<td>Marche</td>
<td>46.5%</td>
<td>14</td>
<td>2,122</td>
<td>88,906</td>
<td>2.4%</td>
</tr>
<tr>
<td>Umbria</td>
<td>35.5%</td>
<td>10</td>
<td>1,257</td>
<td>43,581</td>
<td>2.9%</td>
</tr>
<tr>
<td>Lazio</td>
<td>52.1%</td>
<td>14</td>
<td>5,729</td>
<td>273,343</td>
<td>2.1%</td>
</tr>
<tr>
<td>Abruzzo</td>
<td>44.6%</td>
<td>13</td>
<td>2,413</td>
<td>66,495</td>
<td>3.6%</td>
</tr>
<tr>
<td>Molise</td>
<td>67.1%</td>
<td>4</td>
<td>569</td>
<td>15,194</td>
<td>3.7%</td>
</tr>
<tr>
<td>Campania</td>
<td>55.5%</td>
<td>27</td>
<td>6,720</td>
<td>228,824</td>
<td>2.9%</td>
</tr>
<tr>
<td>Basilicata</td>
<td>57.2%</td>
<td>11</td>
<td>1056</td>
<td>27,335</td>
<td>3.9%</td>
</tr>
<tr>
<td>Apulia</td>
<td>68.4%</td>
<td>16</td>
<td>4,968</td>
<td>195,460</td>
<td>2.5%</td>
</tr>
<tr>
<td>Calabria</td>
<td>66.1%</td>
<td>31</td>
<td>2,848</td>
<td>84,175</td>
<td>3.4%</td>
</tr>
<tr>
<td>Sicily</td>
<td>54.4%</td>
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<td>4,219</td>
<td>217,394</td>
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</tr>
<tr>
<td>Sardinia</td>
<td>56.1%</td>
<td>11</td>
<td>1,500</td>
<td>73,717</td>
<td>2.0%</td>
</tr>
</tbody>
</table>

**Note to Table 1.** Column 1: concealed income as % of taxable income. Column 2: number of tax jurisdictions. Column 3: total number of audits performed. Column 4: number of self-employed individuals. Column 5: total number of audits as % of number of self-employed individuals.
Table 2. Concealed income by sector.

<table>
<thead>
<tr>
<th>Sector of Activity</th>
<th>Evasion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agriculture</td>
<td>44.9</td>
</tr>
<tr>
<td>Handicraft-Food</td>
<td>45.5</td>
</tr>
<tr>
<td>Handicraft- Mining</td>
<td>40.3</td>
</tr>
<tr>
<td>Handicraft- Manufactory</td>
<td>40.5</td>
</tr>
<tr>
<td>Wholesale trade</td>
<td>43.1</td>
</tr>
<tr>
<td>Retail trade</td>
<td>38.6</td>
</tr>
<tr>
<td>Other commercial activities</td>
<td>32.8</td>
</tr>
<tr>
<td>Transports and communication</td>
<td>36.8</td>
</tr>
<tr>
<td>Credit and insurance</td>
<td>17.6</td>
</tr>
<tr>
<td>Other services</td>
<td>42.8</td>
</tr>
<tr>
<td>Professions</td>
<td>14.6</td>
</tr>
<tr>
<td>Non reported</td>
<td>17.4</td>
</tr>
</tbody>
</table>

Note to Table 2. The table reports concealed income as % of taxable income by sector of economic activity.
Table 3. Small and large tax jurisdictions.

<table>
<thead>
<tr>
<th>Region</th>
<th>Jurisdictions</th>
<th></th>
<th></th>
<th>Audits</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td># Small</td>
<td>Large</td>
<td></td>
<td>Small</td>
<td>Large</td>
<td></td>
</tr>
<tr>
<td>Aosta Valley</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>67</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>Piedmont</td>
<td>38</td>
<td>29</td>
<td>9</td>
<td>3,380</td>
<td>3,507</td>
<td></td>
</tr>
<tr>
<td>Lombardy</td>
<td>60</td>
<td>52</td>
<td>8</td>
<td>6,108</td>
<td>6,526</td>
<td></td>
</tr>
<tr>
<td>Friuli-Venezia Giulia</td>
<td>10</td>
<td>7</td>
<td>3</td>
<td>483</td>
<td>924</td>
<td></td>
</tr>
<tr>
<td>Trentino-Alto Adige/Südtirol</td>
<td>12</td>
<td>8</td>
<td>4</td>
<td>440</td>
<td>627</td>
<td></td>
</tr>
<tr>
<td>Veneto</td>
<td>31</td>
<td>25</td>
<td>6</td>
<td>2,239</td>
<td>2,601</td>
<td></td>
</tr>
<tr>
<td>Liguria</td>
<td>10</td>
<td>7</td>
<td>3</td>
<td>1,675</td>
<td>1,986</td>
<td></td>
</tr>
<tr>
<td>Emilia-Romagna</td>
<td>24</td>
<td>19</td>
<td>5</td>
<td>2,876</td>
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<tr>
<td>Tuscany</td>
<td>34</td>
<td>28</td>
<td>6</td>
<td>2,705</td>
<td>2,763</td>
<td></td>
</tr>
<tr>
<td>Marche</td>
<td>14</td>
<td>10</td>
<td>4</td>
<td>1,020</td>
<td>1,102</td>
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</tr>
<tr>
<td>Umbria</td>
<td>10</td>
<td>7</td>
<td>3</td>
<td>464</td>
<td>793</td>
<td></td>
</tr>
<tr>
<td>Lazio</td>
<td>14</td>
<td>11</td>
<td>3</td>
<td>2,441</td>
<td>3,288</td>
<td></td>
</tr>
<tr>
<td>Abruzzo</td>
<td>13</td>
<td>9</td>
<td>4</td>
<td>1,076</td>
<td>1,337</td>
<td></td>
</tr>
<tr>
<td>Molise</td>
<td>4</td>
<td>2</td>
<td>2</td>
<td>193</td>
<td>376</td>
<td></td>
</tr>
<tr>
<td>Campania</td>
<td>27</td>
<td>21</td>
<td>6</td>
<td>3,165</td>
<td>3,555</td>
<td></td>
</tr>
<tr>
<td>Basilicata</td>
<td>11</td>
<td>7</td>
<td>4</td>
<td>472</td>
<td>584</td>
<td></td>
</tr>
<tr>
<td>Apulia</td>
<td>16</td>
<td>11</td>
<td>5</td>
<td>2,291</td>
<td>2,677</td>
<td></td>
</tr>
<tr>
<td>Calabria</td>
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<td>21</td>
<td>10</td>
<td>1,115</td>
<td>1,733</td>
<td></td>
</tr>
<tr>
<td>Sicily</td>
<td>40</td>
<td>33</td>
<td>7</td>
<td>3,572</td>
<td>3,862</td>
<td></td>
</tr>
<tr>
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<td>8</td>
<td>3</td>
<td>696</td>
<td>804</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>412</td>
<td>316</td>
<td>96</td>
<td>36,478</td>
<td>42,385</td>
<td></td>
</tr>
</tbody>
</table>

**Note to Table 3.** The table decomposes the number of jurisdictions and audits by small and large jurisdictions. A small jurisdiction is defined as a jurisdiction whose size in terms of auditable self-employed is below the regional median.